Effects of Solar Radiation Pressure on the Tethered Antenna/Reflector Subsatellite System

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The analytical formulations for the modeling of the solar radiation disturbances on a class of tethered antenna and reflector subsatellite systems are developed based on the assumption that the incident sunlight on the illuminated surface is absorbed completely, reflected completely, or absorbed and reflected in some combination. The effects of the disturbance torque contributed by the shell reflector, the boom, the tether, and the subsatellite are evaluated for stationkeeping operations, and the numerical results show that the tension control law is still able to maintain satisfactory pointing accuracy when the system operates in low or moderate Earth orbit. A hybrid compensation control strategy (involving tether tension plus some kind of actuator attached to the antenna and reflector) might be needed for high Earth orbit, especially the geosynchronous orbit.

Introduction

RECENTLY, a large tethered antenna and reflector subsatellite system (shown in Fig. 1) was proposed for wide-scale communications in which the shallow, spherical, shell-shaped reflector can receive multibeam signals from transmitters on the ground or on the subsatellite and transmit the signals to a variety of small mobile receivers comprising strategic communication links; a second application would involve reflection of high energy beams generated either from the ground facilities or from the subsatellite to preselected targets. A deployable and retrievable tether with a massive subsatellite could be articulated to the tether supporting mechanism, located at the end of a boom that is connected to the apex of the shell reflector, to provide the favorable moment of inertia distribution for overall gravitational stabilization and modulation of the tether tension levels for producing restoring torques on the shallow shell reflector.

A comprehensive survey article was recently published by Misra and Modi.[1] A tether tension control law based on the tether length and length rate for in-plane control was formulated by Rupp.[2] Later on, Liu and Bainum[3] developed the system mathematical model of the antenna and reflector subsatellite system. An optimal control law for tether tension control has been suggested and evaluated. The numerical results show that the transient response can be improved significantly by carefully selecting optimal control gains.

A question arises as to whether or not the tension control laws will still be suitable for maintaining the satisfactory pointing accuracy of the antenna/reflector subsatellite system if some kind of environmental disturbance, say, solar radiation pressure, is considered.

An overview concerning the environmental torques was prepared by Shrivastava and Modi.[4] It is known that the solar radiation disturbance is a dominant environmental disturbance, especially in high Earth orbits, because most other disturbances tend to diminish with increasing altitude. The tension control is based on utilizing the gravitational and centrifugal forces; therefore, the capability of the tension control will be decreased as the altitude level is increased. Thus the solar radiation torque is most likely to be a significant factor in the design of space structures with large surface areas[5] that operate at orbital altitudes above about 1000 km.

The principal source of the radiation force is direct solar illumination. Earth-reflected sunlight and infrared emission from the Earth and its atmosphere are additional sources. The asymmetrical emission from on-board the space structure could be considered as a secondary source of radiation. Major factors in the determination of radiation torques are the intensity and direction of the incident or emitted radiation, the shape of the surface and the location of the illuminated part of the surface with respect to the mass center of the space structure, and the optical properties of the surface on which the radiation is incident or from which it is emitted.

The general formulations for evaluating the resultant solar radiation force and torque are as follows[6]:

$$F_a = -w \int_s (\hat{n} \cdot \hat{\sigma}) \hat{\sigma} \, ds \tag{1}$$

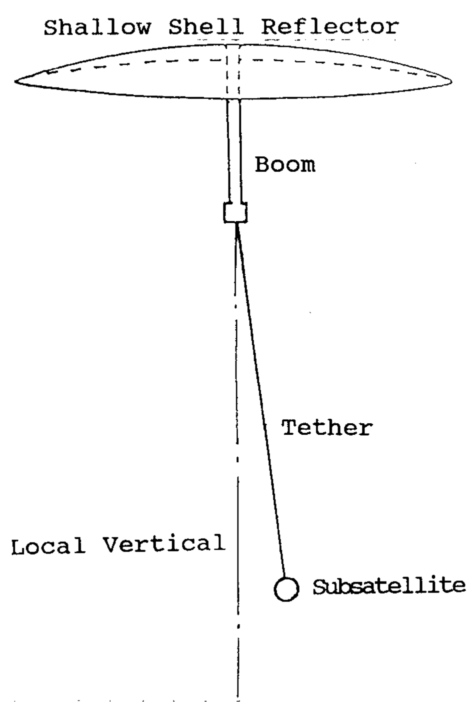

Fig. 1 Tethered antenna/reflector subsatellite system.

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$$F_f = -2w \int_s (\hat{n}\cdot\hat{\sigma})^2 \hat{n}\, ds \tag{2}$$

$$F = (1-\epsilon)F_a + \epsilon F_f \tag{3}$$

$$N_a = -w \int_s (\hat{n}\cdot\hat{\sigma}) r \times \hat{\sigma}\, ds \tag{4}$$

$$N_f = -2w \int_s (\hat{n}\cdot\hat{\sigma})^2 r \times \hat{n}\, ds \tag{5}$$

$$N = (1-\epsilon)N_a + \epsilon N_f \tag{6}$$

where F_a is the resultant solar radiation pressure force on a completely absorbing surface, F_f the resultant solar radiation pressure force on a completely reflecting surface, F the resultant solar radiation pressure force on a general surface with arbitrary reflectivity, N_a the resultant solar radiation pressure torque on a completely absorbing surface, N_f the resultant solar radiation pressure torque on a completely reflecting surface, N the resultant solar radiation pressure torque on a general surface with arbitrary reflectivity, ϵ the reflection coefficient (i.e., the ratio of the energy density of the reflected light flux to the energy density of the incident light flux), s the illuminated part of the surface, w the solar constant, $\hat{n}$ the unit outward normal vector of the differential surface element, $\hat{\sigma}$ the unit vector opposite to the incident sunlight, and r the position vector of the differential surface element from the center of mass of the system. Similar expressions have been given by Modi and Pande[7] as well as Modi and Kumar.[8]

A major difficulty in the modeling of solar radiation torques acting on the space structure lies in the determination of the boundary of the illuminated area on the surface of the orbiting structure. In general, the boundary varies with the position of the orbiting structure in the orbit and the attitude and the shape of the structure.

Sometimes it is extremely difficult to obtain an exact analytical formulation for the solar radiation disturbance. Bainum and Krishna[9] used a numerical approach to approximate the force and moment induced by the solar radiation pressure on an orbiting flexible beam and plate.

One of the objectives of this paper is to develop the analytical formulation for the modeling of the solar radiation disturbance on a class of large tethered antenna/reflector types of orbiting structures. The other is to evaluate the effects of these disturbances on stationkeeping and to determine by numerical simulation at which altitude level the tension control law is still able to maintain the satisfactory pointing accuracy of the tethered antenna and reflector subsatellite system. Because of the altitudes under consideration for this system, the effects of other disturbances such as those due to aerodynamics, magnetic field, etc., are not considered here.

Modeling of the Solar Radiation Disturbance

In practice, the properties of the surface are rarely known in sufficient detail to evaluate the required functions; therefore, in this paper the assumption is made that all incident radiation is completely absorbed, or specularly reflected, or absorbed and reflected in some combination. In addition to this, for mathematical modeling of the solar radiation disturbance, the following assumptions are made: 1) The reflector and the boom are assumed to be rigid bodies, the subsatellite is assumed a rigid sphere, and the tether is assumed to be compressible but considered as a straight line (neglecting the transverse deformation) during the swing motion. The surfaces of these bodies are characterized by isotropic reflectivity properties. 2) The spherical, shallow shell is totally illuminated either on the inner shell surface or the outer shell surface, because the ratio of the height of the shell to the radius of the base of the shell is far less than unity. 3) Two phenomena related to local shadowing on the tether and the boom are considered. One is that some sections of the tether or the boom will not be illuminated in certain positions of the orbit due to the shadow of the reflector. The other is that the boom or the tether is considered as a cylindrical column; therefore, at any instant of time the tether or the boom can only be illuminated on one side when the emission of the sunlight and the reflected sunlight from the Earth or other space objects are neglected. In addition, the shadow of the Earth on the orbit has been taken into account during the numerical simulation. 4) The solar incidence vector in the orbital coordinate reference system is treated as a time-varying parameter. It varies with the inclination of the orbit and the position of the space structure in the orbit. A circular and equatorial orbit is assumed in the numerical simulation without losing generality. 5) The center of mass of the system is assumed to be at the center of mass of the reflector because the preliminary simulation showed that the effects of the center of mass shift can be neglected for the parameters assumed here.

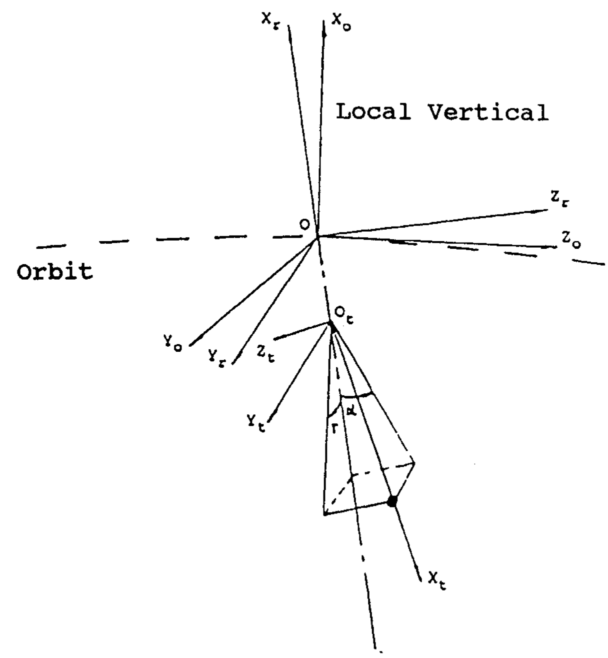

Fig. 2 Coordinate reference systems.

The coordinate systems used in the development of the system equations of motion are shown in Fig. 2; $OX_oY_oZ_o$ is an orbital reference system centered at the center of mass of the shell reflector with OX_o along the local vertical, OY_o along the normal to the orbit plane, and OZ_o along the orbital tangent velocity direction. $OX_rY_rZ_r$ is a shell body reference frame; the axes OX_r, OY_r, and OZ_r are principal axes of the shell reflector. $O_tX_tY_tZ_t$ is the tether reference frame with O_tX_t along the undeformed tether, where O_t is the point from which the tether is deployed or retrieved. The coordinates of O_t in the shell reference frame are $(h_x, 0, 0)$ where h_x has a negative value.

The yaw, pitch, and roll angles of the shell are denoted by ψ, θ, and ϕ, respectively. An Euler angle rotation sequence is assumed as ψ about the local vertical, then θ about the orbit normal, and finally, ϕ about the tangent of the orbit. Therefore, the transformation from $OX_oY_oZ_o$ to $OX_rY_rZ_r$ is given by

$$[X_r, Y_r, Z_r]^T = Q_r[X_o, Y_o, Z_o]^T \tag{7}$$

$$Q_r = \begin{bmatrix} c\phi c\theta & s\phi c\psi + c\phi s\theta s\psi & s\phi s\psi - c\phi s\theta c\psi \\ -s\phi c\theta & c\phi c\psi - s\phi s\theta s\psi & c\phi s\psi + s\phi s\theta c\psi \\ s\theta & -c\theta s\psi & c\theta c\psi \end{bmatrix} \tag{8}$$

The in-plane swing angle of the tether α depicts the angle between the projection of the nondeformed tether in the orbit plane and the OX_r axis; and the out-of-plane swing angle of the tether γ indicates the angle between the projection of the undeformed tether in the OX_rY_r plane and the OX_r axis. The transformation from $O_tX_tY_tZ_t$ to $OX_rY_rZ_r$ can be derived by

$$[X_r,\ Y_r,\ Z_r]^T = Q_t^{-1}[X_t,\ Y_t,\ Z_t]^T + [h_x,\ 0,\ 0]^T \tag{9}$$

$$Q_t = \begin{bmatrix} -c\gamma c\alpha & s\gamma & s\alpha c\gamma \\ s\gamma c\alpha & c\gamma & -s\alpha s\gamma \\ -s\alpha & 0 & -c\alpha \end{bmatrix} \tag{10}$$

The unit vector opposite to the solar incidence can be formed in the orbital reference system:

$$\hat{\sigma}_o = [a_o,\ b_o,\ c_o]^T \tag{11}$$

where a_o, b_o, and c_o are components along the OX_o, OY_o, and OZ_o axes, respectively. Furthermore, the unit vector opposite to the solar incidence can be transformed into the shell reference system:

$$\hat{\sigma}_r = [a_r,\ b_r,\ c_r]^T = Q_r[a_o,\ b_o,\ c_o]^T \tag{12}$$

Here a_r, b_r, and c_r represent the components along the OX_r, OY_r, and OZ_r axes, respectively. Similarly, this vector in the tether reference system can be written as:

$$\hat{\sigma}_t = [a_t,\ b_t,\ c_t]^T = Q_t[a_r,\ b_r,\ c_r]^T \tag{13}$$

where a_t, b_t, and c_t are the components along the O_tX_t, O_tY_t, and O_tZ_t axes, respectively.

The disturbance torque formed by the solar radiation pressure on the system is contributed by the reflector, boom, tether, and subsatellite.

Disturbance Torque Induced by the Solar Radiation Pressure on the Antenna/Reflector

The solar radiation pressure on the surface of a differential element in the reflector and boom can be denoted as $\mathrm{d}\boldsymbol{f}$, and the position vector of this differential element with respect to the center of mass of the reflector can be defined as $\boldsymbol{r}$; therefore, the differential torque due to the solar radiation pressure on this differential element is $\mathrm{d}N = \boldsymbol{r} \times \mathrm{d}\boldsymbol{f}$, and the integration of all of these differential torques on the reflector and boom will result in the solar radiation disturbance torque contributed by the reflector and the boom. The area of the integration is determined by the illumination boundary condition:

$$(\hat{n}\cdot\hat{\sigma}) \geq 0 \tag{14}$$

For a completely absorbing reflector surface

$$N_{ra} = [N_{rax},\ N_{ray},\ N_{raz}]^T = R_a[0,\ c_r,\ -b_r]^T \tag{15}$$

For a completely reflecting reflector surface

$$N_{rf} = [N_{rfx},\ N_{rfy},\ N_{rfz}]^T = R_f[0,\ c_r,\ -b_r]^T \tag{16}$$

For a reflector surface with arbitrary reflectivity properties

$$N_r = [N_{rx},\ N_{ry},\ N_{rz}]^T = (1-\epsilon)N_{ra} + N_{rf} \tag{17}$$

where

$$R_a = -\pi w R^3 c_r \sin^2\beta_0\left(1 - \cos\beta_0 - \frac{r_0}{R}\right)$$

$$R_f = -\pi w R^3 c_r \sin^4\beta_0\left(1 - \frac{r_0}{R}\right)$$

and where w is the solar constant, β_0 the cone angle of the spherical shell reflector, R the radius of the spherical shell reflector, and r_0 the distance from the apex of the shell to the center of mass of the shell.

Disturbance Torque Induced by the Solar Radiation Pressure on the Boom

For a completely absorbing boom surface

$$N_{ba} = [N_{bax},\ N_{bay},\ N_{baz}]^T = B_a[0,\ -c_r,\ b_r]^T \tag{18}$$

For a completely reflecting boom surface

$$N_{bf} = \begin{bmatrix} N_{bfx} \\ N_{bfy} \\ N_{bfz} \end{bmatrix} = B_f \begin{bmatrix} 0 \\ -\left[\frac{b_r^3}{3}\sin^3\lambda_b + \frac{2}{3}b_r c_r \cos^3\lambda_b + c_r^2\left(\sin\lambda_t - \frac{\sin^3\lambda_b}{3}\right)\right] \\ b_r^2\left(\cos\lambda_b - \frac{\cos^3\lambda_b}{3}\right) + \frac{2}{3}b_r c_r \sin^3\lambda_b + \frac{c_r^2}{3}\cos^3\lambda_b \end{bmatrix} \tag{19}$$

where

$$\lambda_b = \begin{cases} \tan^{-1}\dfrac{c_r}{b_r} & (b_r > 0) \\ \tan^{-1}\dfrac{c_r}{b_r} + \pi & (b_r < 0) \end{cases}$$

For a boom surface with arbitrary reflectivity properties

$$N_b = [N_{bx},\ N_{by},\ N_{bz}]^T = (1-\epsilon)N_{ba} + \epsilon N_{bf} \tag{20}$$

where

$$B_a = -w r_b(b_r \cos\lambda_b + c_r \sin\lambda_b)(l_b^2 - l_{b0}^2)$$

$$B_f = -2 w r_b(l_b^2 - l_{b0}^2)$$

and r_b is the radius of the cylindrical column boom, l_b the length of the boom, and l_{b0} the length of the shadowed boom.

Disturbance Force Induced by the Solar Radiation Pressure on the Spherical Subsatellite

It is known that the resultant force of the solar radiation pressure on a sphere is independent of the surface properties and acts at the center of this sphere.[6]

For both absorbing and reflecting subsatellite surfaces

$$F_{sa} = [F_{sax},\ F_{say},\ F_{saz}]^T = \pi w r_s^2[a_t,\ b_t,\ c_t]^T$$
$$= F_{sf} = [F_{sfx},\ F_{sfy},\ F_{sfz}]^T \tag{21}$$

Here r_s is the radius of the spherical subsatellite.

Disturbance Torque Induced by the Solar Radiation Pressure on the Subsatellite About the Point O_t

For both completely absorbing and reflecting subsatellites

$$N_{sa} = [N_{sax},\ N_{say},\ N_{saz}]^T = l_t w \pi r_s^2[0,\ -c_t,\ b_t]^T$$
$$= N_{sf} = [N_{sfx},\ N_{sfy},\ N_{sfz}]^T \tag{22}$$

where l_t is the instantaneous length of the tether.

Disturbance Force Induced by the Solar Radiation Pressure on the Tether

For a completely absorbing tether surface

$$F_{ta} = [F_{tax}, F_{tay}, F_{taz}]^T$$
$$= 2wr_t(b_t \cos\lambda_t + c_t \sin\lambda_t)(l_t - l_{t0})[a_t, b_t, c_t]^T \tag{23}$$

For a completely reflecting tether surface

$$F_{tf} = \begin{bmatrix} F_{tfx} \\ F_{tfy} \\ F_{tfz} \end{bmatrix} = 4w\pi(l_t - l_{t0})$$
$$\times \begin{bmatrix} 0 \\ b_t^2\left(\cos\lambda_t - \dfrac{\cos^3\lambda_t}{3}\right) + \dfrac{c_t^2}{3}\cos^3\lambda_t + \dfrac{2}{3}b_t c_t \sin^3\lambda_t \\ \dfrac{b_t^2}{3}\sin^3\lambda_t + c_t^2\left(\sin\lambda_t - \dfrac{\sin^3\lambda_t}{3}\right) + \dfrac{2}{3}b_t c_t \cos^3\lambda_t \end{bmatrix} \tag{24}$$

where

$$\lambda_t = \begin{cases} \tan^{-1}\dfrac{c_t}{b_t} & (b_t > 0) \\ \tan^{-1}\dfrac{c_t}{b_t} + \pi & (b_t < 0) \end{cases}$$

For a tether surface with arbitrary reflectivity properties

$$F_t = [F_{tx}, F_{ty}, F_{tz}]^T = (1-\epsilon)F_{ta} + \epsilon F_{tf} \tag{25}$$

where r_t is the radius of the tether and l_{t0} the length of the shadowed tether.

Disturbance Torque Induced by Solar Radiation Pressure on the Tether About the Point O_t

For a completely absorbing tether surface

$$N_{ta} = [N_{tax}, N_{tay}, N_{taz}]^T$$
$$= 2wr_t(b_t \cos\lambda_t + c_t \sin\lambda_t)(l_t^2 - l_{t0}^2)[0, -c_t, b_t]^T \tag{26}$$

For a completely reflecting tether surface

$$N_{tf} = \begin{bmatrix} N_{tfx} \\ N_{tfy} \\ N_{tfz} \end{bmatrix} = 4wr_t(l_t^2 - l_{t0}^2)$$
$$\times \begin{bmatrix} 0 \\ -\left[\dfrac{b_t^2}{3}\sin^3\lambda_t + c_t^2\left(\sin\lambda_t - \dfrac{\sin^3\lambda_t}{3}\right) + \dfrac{2}{3}b_t c_t \cos^3\lambda_t\right] \\ b_t^2\left(\cos\lambda_t - \dfrac{\cos^3\lambda_t}{3}\right) + \dfrac{c_t^2}{3}\cos^3\lambda_t + \dfrac{2}{3}b_t c_t \sin^3\lambda_t \end{bmatrix} \tag{27}$$

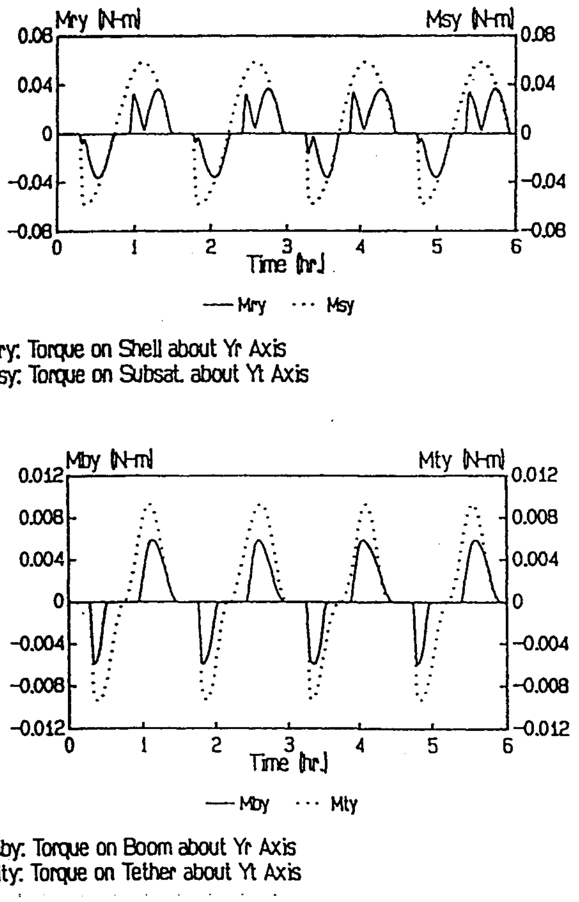

Fig. 3 In-plane solar radiation torques (completely absorbing surface).

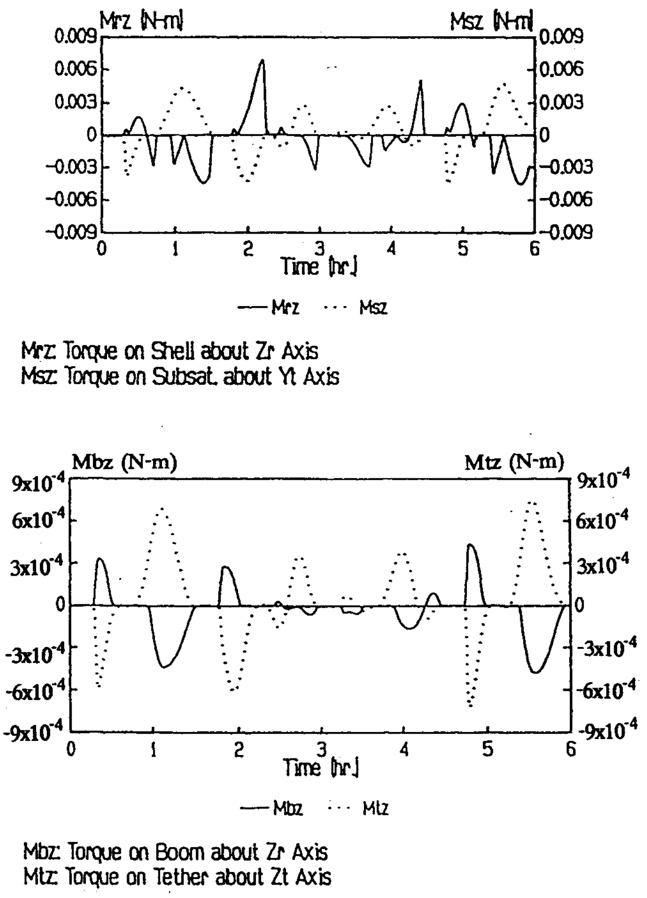

Fig. 4 Out-of-plane solar radiation torques (completely absorbing surface).

For a tether surface with arbitrary reflectivity properties

$$N_t = [N_{tx}, N_{ty}, N_{tz}]^T = (1-\epsilon)N_{ta} + \epsilon N_{tf} \tag{28}$$

Modeling of the System Equations of Motion

Because the preceding analytical models of the solar radiation disturbance are derived without taking the vibrations of the tether and the shell into consideration, the system equations of motion must also be developed based on this assumption.

Besides, it is known that the roll and yaw motion of the shell as well as the out-of-plane swing motion of the tether are decoupled from the pitch motion of the shell and the in-plane swing motion of the tether in the linear range.[3] Thus, after placing the solar radiation disturbances just derived into the right-hand side of the equations of motion and proceeding with a series of complicated algebraic manipulations, the linear nondimensional equations for in-plane motion are obtained:

$$\theta'' + k_2\alpha'' - 3\Omega_y^*\theta - k_5\epsilon_t' = \frac{1}{\omega_c^2 J_y^*}\left\{N_{py} + N_{by} + h_x\left[F_{sv} + F_{tv} - \alpha(F_{sx} + F_{tx})\right]\right\} \tag{29}$$

$$k_1\alpha'' + \theta'' + 3\theta + 3\alpha - k_4\epsilon_t' = \frac{N_{tv} + N_{sv}}{\omega_c^2(H_{xx} - h_x I_x)} \tag{30}$$

$$\epsilon_t'' + 2(k+\beta_x)\theta' - 3k\epsilon_t + 2k\alpha' = 3(k+\beta_x) + \frac{T_x}{l_{tc}\omega_c^2 m_{st}} + \frac{F_{sx} + F_{tx}}{l_{tc}\omega_c^2 m_{st}} \tag{31}$$

where

$$m_{st} = m_s + m_t, \quad k = \frac{m_s + (m_t/2)}{m_{st}}, \quad k_1 = \frac{H_{xx}}{H_{xx} - I_x h_x}$$

$$k_2 = -I_x\frac{h_x}{J_x^*}, \quad k_4 = \frac{2I_x l_{tc}}{H_{xx} - h_x I_x}, \quad k_5 = -2m_{st}h_x\frac{l_{tc}}{J_y^*}$$

$$I_x = \int_{s,t} x\,\mathrm{d}m, \quad H_{xx} = \int_{s,t} x^2\,\mathrm{d}m, \quad J_x^* = J_x, \quad \beta_x = -\frac{h_x}{l_{tc}}$$

$$J_y^* = J_y + h_x^2 m_{st} - I_x h_x, \quad J_z^* = J_z + h_x^2 m_{st} - I_x h_x$$

$$\Omega_y^* = \frac{J_x^* - J_z^*}{J_y^*}$$

and $(\ )' = \mathrm{d}(\ )/\mathrm{d}\tau$, $(\ )'' = \mathrm{d}(\ )'/\mathrm{d}\tau$, $\tau = \omega t$, $\epsilon_t = \Delta l_t / l_{tc}$, and $\Delta l_t = l_t - l_{tc}$. T_x is the tension of the tether, ω_c the orbital angular velocity, m_s the mass of the subsatellite, m_t the mass of the tether, l_{tc} the commanded length of the tether, m_{st} the total mass of the subsatellite and the tether, h_x the coordinate of the length of the boom in the shell reference system, and J_x, J_y, and J_z the principal moments of inertia of the shell.

If we let

$$\Delta f = 3(k+\beta_x) + \frac{T_x}{l_{tc}\omega_c^2 m_{st}}, \quad D_\alpha = \frac{N_{tv} + N_{sv}}{\omega_c^2(H_{xx} - h_x I_x)}$$

$$D_\epsilon = \frac{F_{sx} + F_{tx}}{l_{tc}\omega_c^2 m_{st}}$$

$$D_\theta = \frac{N_{py} + N_{by} + h_x\left[F_{sv} + F_{tv} - \alpha(F_{sx} + F_{tx})\right]}{\omega_c^2 J_y^*}$$

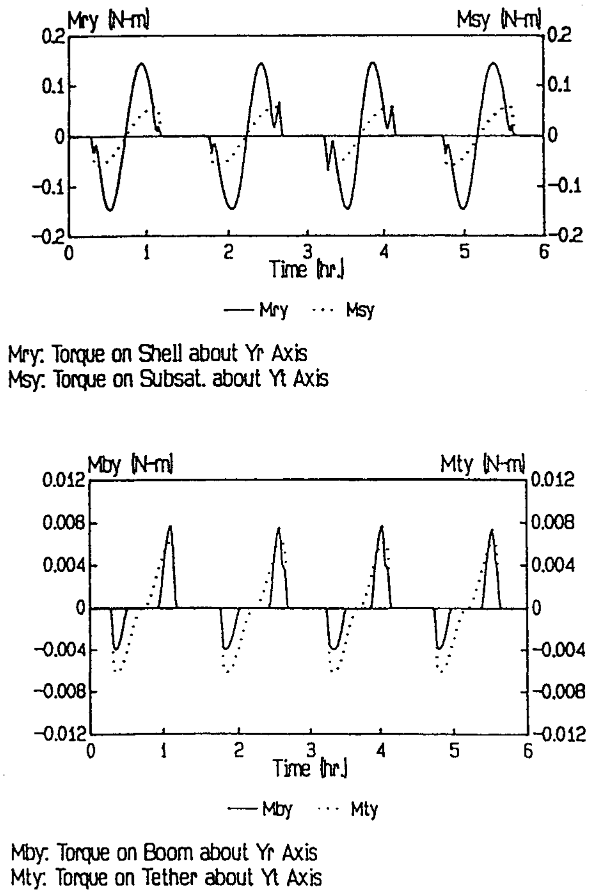

Fig. 5 In-plane solar radiation torques (completely reflecting surface).

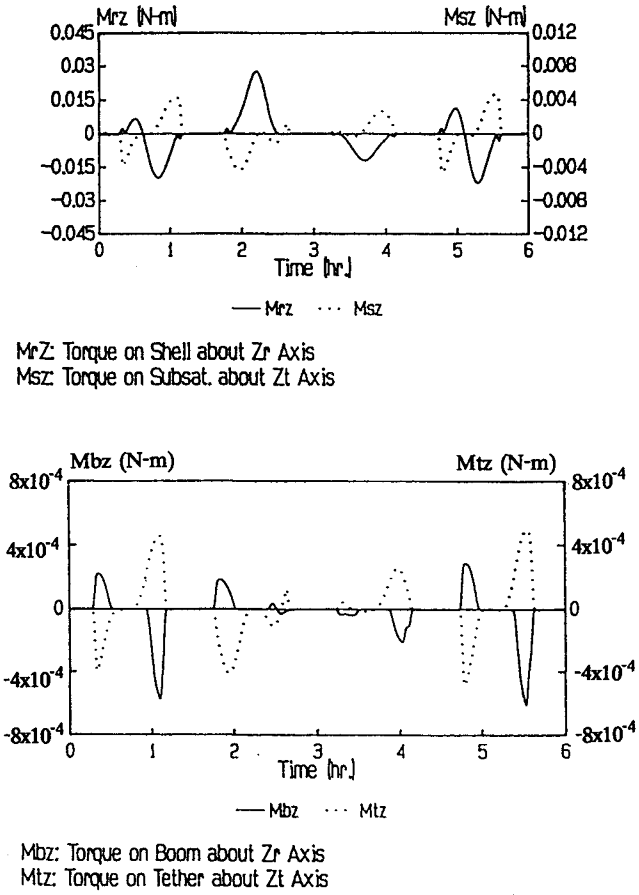

Fig. 6 Out-of-plane solar radiation torques (completely reflecting surface).

then the equations of in-plane motion become:

$$k_2\alpha'' + \theta'' - 3\Omega_y^{*}\theta - k_5\epsilon_t' = D_\theta \tag{32}$$

$$k_1\alpha'' + \theta'' + 3\alpha + 3\theta - k_4\epsilon_t' = D_\alpha \tag{33}$$

$$\epsilon_t'' + 2(k+\beta_x)\theta' + 2k\alpha' - 3k\epsilon_t = \Delta f + D_\epsilon \tag{34}$$

The system equations of motion can be written in matrix form as:

$$x' = Ax + Bu + CD \tag{35}$$

where

$$x = [\theta, \alpha, \epsilon, \theta', \alpha', \epsilon']^T$$

and

$$A = \begin{bmatrix} O & I \\ A_1 & A_2 \end{bmatrix}, \quad B = \begin{bmatrix} O \\ B_1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

It is obvious that if we consider:

$$u = u_o + u_c, \quad u_o = \Delta f_o, \quad u_c = \Delta f_c$$

and if u_c can be found such that

$$Bu_c + CD = O$$

then the system equation (35) becomes

$$x' = Ax + Bu_o = Ax + B\Delta f_o \tag{36}$$

and an all-state feedback control law can be constructed:

$$\Delta f_o = -Kx \tag{37}$$

For the system parameters $m_r = 10{,}000$ kg, $m_s = 500$ kg, $m_t = 8.35$ kg, $l_{tc} = 1000$ m, and an 80-m boom, an optimal control u_o that minimizes the performance index

$$J = \int_0^\infty (x^TQx + u_o^TRu_o)\,dt \tag{38}$$

can be found by carefully selecting Q and R. Here x is the state variable, Q the positive semidefinite state penalty matrix, and R the positive definite control penalty matrix.

Some typical simulation results demonstrate that the transient response with the optimal gains obtained by suitable Q and R has low overshoots and short settling and rising times. The discussion about how the structural parameters affect the overall stability has been made by Liu and Bainum.[3]

But the optimal control based only on the use of tether tension cannot realize $Bu_c + CD = 0$, since only one control exists. Thus, the best way to proceed is to form a least squares compensator:

$$\Delta f_c = -(B^TB)^{-1}B^TCD \tag{39}$$

Nevertheless, the disturbance D_θ and D_α cannot be totally compensated for in this way. Fortunately, the effects of D_θ and D_α are not significant in lower orbit. On the other hand, the in-plane motion of the shell and the tether are coupled with the longitudinal motion of the tether. The system can still be controlled to a satisfactory degree in lower orbit, which can be demonstrated by some numerical simulation results (see Fig. 7). If the altitude level is increased to approximately 8600 km, it is difficult to maintain the satisfactory pointing accuracy of the shell reflector because of the significant effects of

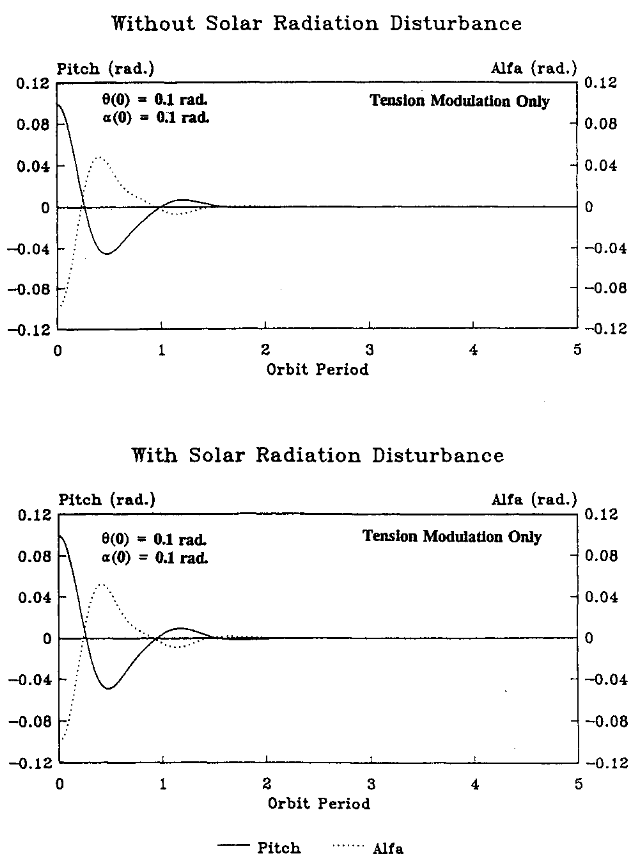

Fig. 7 Tether control response (LEO).

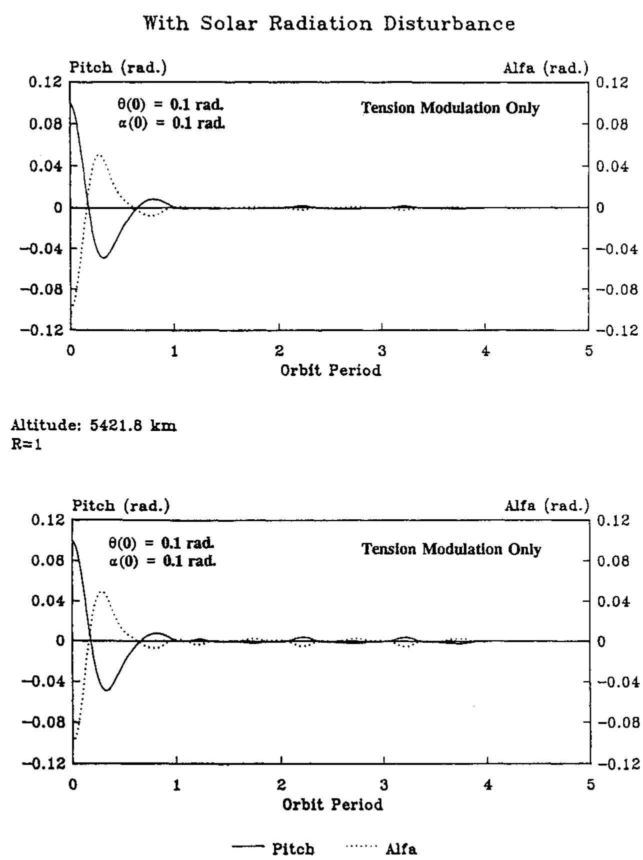

Fig. 8 Tether control response (HEO).

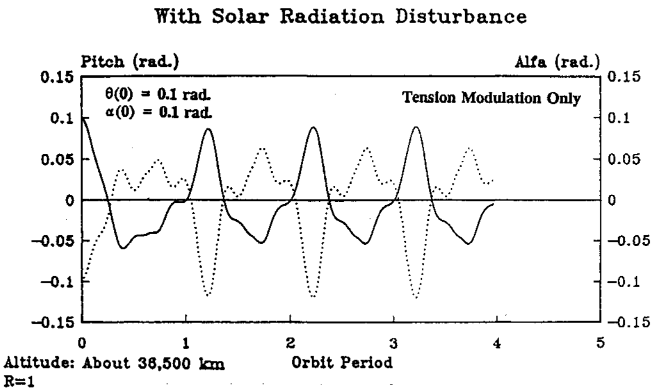

Fig. 9a Tether control response (GEO).

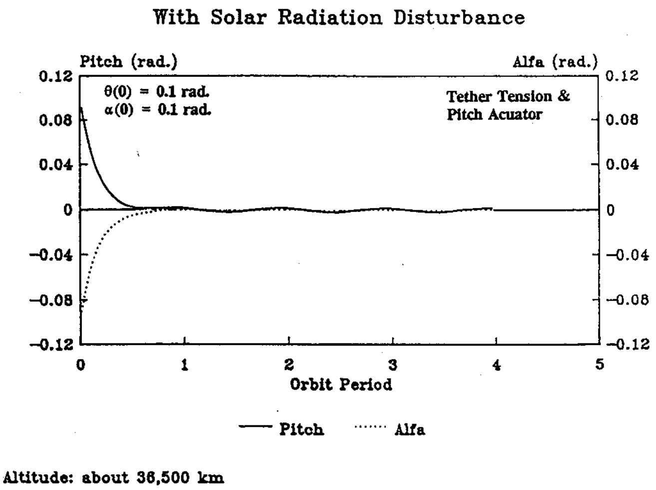

Fig. 9b Tether control response (GEO).

the uncompensated disturbances. Therefore, to retain control of the system in higher orbit with the presence of the solar radiation disturbances, a hybrid control system, i.e., the introduction of some other kind of actuator (e.g., a thruster) on the shell or on the subsatellite, is necessary.

Numerical Simulations

The numerical simulations are conducted based on the preceding assumptions. Since the solar incidence vector expressed in the orbital reference system is a function of the orbital period and the motions of the tethered system are varied periodically for the most part, the disturbance torques induced by the solar radiation pressure are periodic functions; and the period is close to the orbital period, which is demonstrated by the numerical simulation results (see Figs. 3–6). Comparing Figs. 3–6, we can easily see that the magnitude of in-plane torques is much larger than those of the out-of-plane torques, and the maximum amplitude of the in-plane torques in the completely reflecting case is greater than that in the completely absorbing case. In the completely absorbing case, the torque acting on the subsatellite about the Y_t axis is larger than the torque acting on the shell about the Y_r axis, but vice versa in the completely reflecting case. Therefore, if the solar radiation disturbances are considered, the major disturbances are the in-plane torques acting on the shell and the subsatellite because of the large illuminated area on the shell and the long moment arm from the center of mass of the whole system to the subsatellite.

From Fig. 7, it is noticed that the effects of the solar radiation disturbances are small in low Earth orbit (LEO), since the steady-state performance of the tethered system controlled by the tether tension with the solar radiation disturbances is almost the same as that without the solar radiation disturbances. But in high Earth orbit (HEO), the higher the altitude level is, the worse the steady-state performance of the tether system (see Fig. 8); therefore, the effects cannot be neglected, especially, when the altitude level is higher than about 8600 km. This suggests that the attitude and dynamic control law for such a system in HEO or geosynchronous orbit (GEO) should involve some kind of active (actuator) control on the shell or subsatellite. Figure 9 is a demonstration of the use of a hybrid control system (pitch thruster) for the in-plane motion of the antenna (reflector) system. For the hybrid control system results illustrated (and others not shown) about 80% of the total control effort must be provided by the actuator.

Conclusions

Several conclusions can be obtained from the numerical simulation results: The analytical modeling of solar radiation effects, introduced here, can be extended to other large space systems. The effects of the solar radiation disturbance in lower orbit can be neglected but they are more significant in higher orbits. The tension control law with optimal control gains that is obtained by carefully selecting the state penalty matrix Q and control penalty matrix R is able to maintain the satisfactory pointing accuracy when the altitude is lower than about 8634 km for a 100-m-diam shell reflector connected to a 1-km tether at the end of an 80-m boom. The maximum magnitude of the solar radiation torque that disturbs the in-plane motion of the system is larger than that which disturbs the out-of-plane motion of the system. The dominant contribution to the solar disturbance torque is induced by the solar radiation pressure on the shell reflector or the subsatellite. In the case of a completely absorbing surface, i.e., $\epsilon = 0$, the magnitude of the disturbance torque is smaller than that in the case of a completely reflecting surface, i.e., $\epsilon = 1$. In the case of a completely absorbing surface, the solar radiation disturbance torque contributed by the subsatellite is greater than that contributed by the reflector shell, and vice versa in the case of a completely reflecting surface. To control such a system to a satisfactory degree, a hybrid control law based on both tension modulation and some additional active control (actuator) is needed.

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